SoSe 26 Computer Graphics Solutions

Igor Dimitrov

2026-04-21

Table of contents

Preface

This is a Quarto book.

To learn more about Quarto books visit https://quarto.org/docs/books.

1 Sheet 1

Cemile Buyukakkac
Malte Herzog
Igor Dimitrov

1.1 Assignment 1.1

a) |Pixel| = 3840 x 2160 = 829440
The whole dispaly is generated in $t = \frac{1}{f} = \frac{1}{60}s$
$\Rightarrow$ Generation of a single pixel can take at most:

$$t_{max} = \frac{1}{3840 \times 2160 \times 60} s \approx 2ns$$

b) Data load of 24 bits per pixel is due to three color channels Red, Green, and Blue, each 8 bit , i.e. 1B. Therefore there are 8 Bytes per pixel and 3840 x 2160 pixels in total. Since the display is regenerated every 1/60 seconds, we have:

$$\begin{aligned} B &= 3B \times 3840 \times 2160 \times 60 \frac{1}{s} \\ &\approx 1.4GB/s \end{aligned}$$

1.2 Assignment 1.2

a) There are three types of light-sensitive receptors,

- S: corresponding to blue
- M: corresponding to green
- L corresponding to red but the perception of color is calculated simply as superposition of three channels but rather as two channels combination of pairs of opposing colors

b) To focus (or zoom in this context) means to reduce the field of vision, and reducing the field of vision corresponds to a smaller solid angle. The smaller the solid angle is, the less light arrives at the camera lens or human eye through the telescope, which makes the object appear dimmer, or not bright enough.

c) metamers are color pairs that look or are perceived identically to the human eye, although they emit different light spectra, because they produce the same response in the three cone types (L, M, S)

2 Sheet 2

2.1 Assignment 1

a)
- Vector-based advantages:
  - resolution independent
- Vector-based disadvantages:
  - image build-up is difficult in complex scenes
- Raster-based advantages:
  - repeated image build-up independent of scene complexity
- Raster-based disadvantages:
  - Moire effects due to finite number of pixels

b) a spectral color, or a monochromatic color, is light produced by the light of single wavelength in the visible spectrum

c)

d) mach bands are an optical illusion where we perceive extra-bright and extra-dark bands near the boundary between two regions with different luminance gradients

a receptive field have usually a center-surround structure where light in the center excites a neuron, while the light in the surrounding region inhibits it, or vice versa.

Mach bands arise because these center-surround receptive fields perform a local constrast enhancement. Near a luminance edge, one receptive field may receive more excitation than inhibition, making the side look brither, or vice versa. e) three operations: i)

relation to YIQ

in the YIQ model the Y channel is calculated approximately as:

Y = 0.3 R + 0.6G + 0.1 B

where we see that green contributes the most and blue the least. This corresponds roughly to the human perception of brightness - it is strongest around green-yellow region and weakeste in the blue region, due to S cells contributing close to nothing to the perception of brightness.

I and Q channels carry color-difference information. They do not correspond exactly to the biological opponent channels, but they similarly separate color information from brithness detail - human eye is much more sensitive to contrast than to color difference

l) The light spectrum is given as

$$P(\lambda) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\lambda-\mu)^2}{2\sigma^2}\right) W$$

Is a Guassian / normal distribution shape:

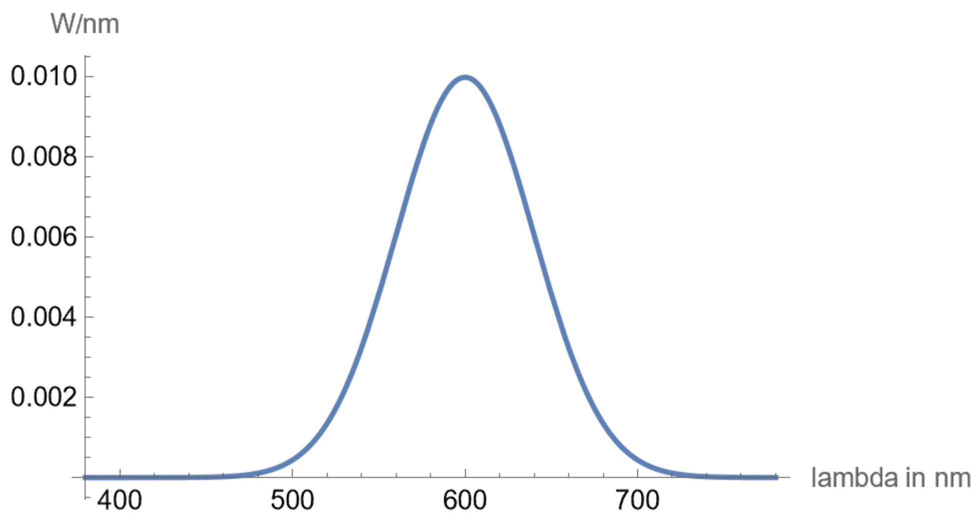

Figure 2.1: $P(\lambda)$

And the color matching functions are given as:

$$\bar{x}(\lambda) = 1.065 \exp\left(-\frac{1}{2}\left(\frac{\lambda - 595.8}{33.33}\right)^2\right) + 0.366 \exp\left(-\frac{1}{2}\left(\frac{\lambda - 446.8}{19.44}\right)^2\right)$$

$$\bar{y}(\lambda) = 1.014 \exp\left(-\frac{1}{2}\left(\frac{\ln\lambda - \ln 556.3}{0.075}\right)^2\right)$$

$$\bar{z}(\lambda) = 1.839 \exp\left(-\frac{1}{2}\left(\frac{\ln\lambda - \ln 449.8}{0.051}\right)^2\right)$$

which can be plotted as follows (together with a scaled version of $P(\lambda)$):

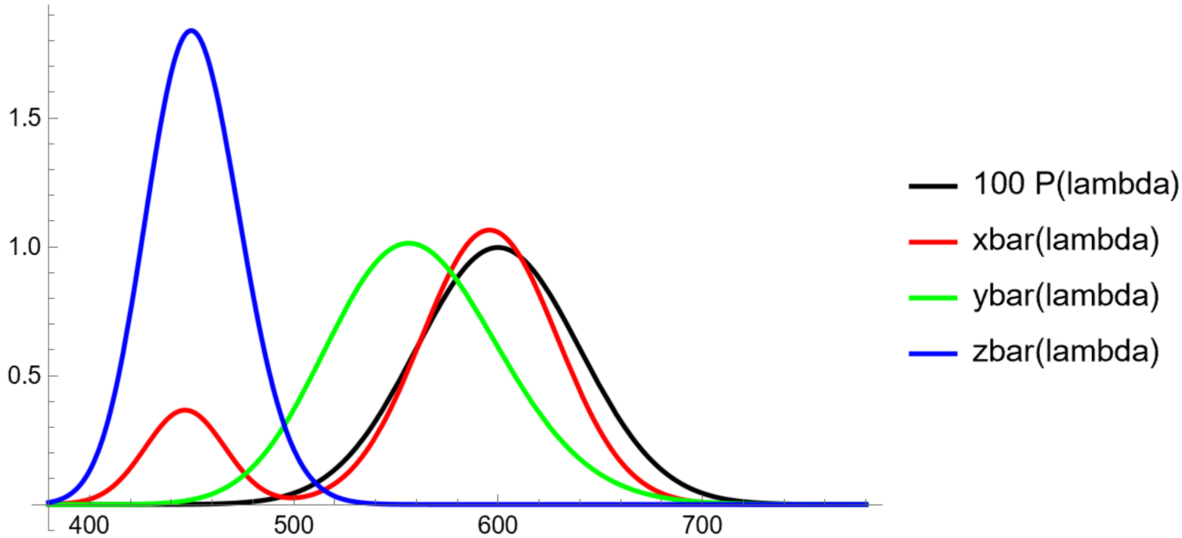

Figure 2.2: x, y, z

To compute X, Y, Z we compute the integrals:

$$X = \int_\lambda \bar{x}(\lambda)\, P(\lambda)\, d\lambda,$$
$$Y = \int_\lambda \bar{y}(\lambda)\, P(\lambda)\, d\lambda,$$
$$Z = \int_\lambda \bar{z}(\lambda)\, P(\lambda)\, d\lambda.$$

We can use a symbolic or numerical mathematics software. In our case we used mathematica where we computed the integrals with the following code:

First we define the functions:

```
sigma = 40;
mu = 600;

P[lambda_] := 1/Sqrt[2 Pi sigma^2] Exp[-(lambda - mu)^2/(2 sigma^2)];

xbar[lambda_] :=
  1.065 Exp[-1/2 ((lambda - 595.8)/33.33)^2] +
  0.366 Exp[-1/2 ((lambda - 446.8)/19.44)^2];

ybar[lambda_] :=
```

```
  1.014 Exp[-1/2 ((Log[lambda] - Log[556.3])/0.075)^2];

zbar[lambda_] :=
  1.839 Exp[-1/2 ((Log[lambda] - Log[449.8])/0.051)^2];
```

Then we compute the integrals with:

```
X = NIntegrate[xbar[lambda] P[lambda], {lambda, 380, 780}];

Y = NIntegrate[ybar[lambda] P[lambda], {lambda, 380, 780}];

Z = NIntegrate[zbar[lambda] P[lambda], {lambda, 380, 780}];
```

and find

$$X \approx 0.679966$$
$$Y \approx 0.569252$$
$$Z \approx 0.00561767$$

and finally we compute the x, y values as:

$$x = \frac{X}{X+Y+Z} \approx 0.541881$$
$$y = \frac{Y}{X+Y+Z} \approx 0.453642$$

To compute linear RGB from XYZ we use the following linear transformation:

$$\begin{pmatrix} R_{\text{lin}} \\ G_{\text{lin}} \\ B_{\text{lin}} \end{pmatrix} = \begin{pmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}.$$

We achieve this with the following mathematica code:

```
XYZtoLinearSRGB[{X_, Y_, Z_}] := {
   3.2406 X - 1.5372 Y - 0.4986 Z,
  -0.9689 X + 1.8758 Y + 0.0415 Z,
   0.0557 X - 0.2040 Y + 1.0570 Z
}

{R, G, B} = XYZtoLinearSRGB[{X, Y, Z}]
```

and obtain:

$$
\begin{aligned}
R &\approx 1.33 \\
G &\approx 0.41 \\
B &\approx -0.07
\end{aligned}
$$

Here R and B are outside of representable RGB gamut. Clipping them we get

$$
\begin{aligned}
R &\approx 1 \\
G &\approx 0.41 \\
B &\approx 0
\end{aligned}
$$

Converting this to $[0, 255]$ scale we get

$$(R, G, B) = (255, 171, 0)$$

This is a bright yellow-orange color, with no blue