WS 23/24 Numerics Notes

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2023-10-30

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Preface

Notes for the lecture "WS 23/24 Numerics 0" at Uni Heidelberg.

1 Floating Point Numbers

1.1 ANSI/IEEE 64 Bit

Let $\tilde{a}$ be a 64 bit IEEE floating point number. $\tilde{a}$ is represented as

```
S E ... E M ... M
```

Where S is the **sign** bit, 11 E's are the **exponent** bits and 52 M's are **mantissa** bits. Interpretation (Case analysis on value of E):

1. $\texttt{E} = 0$, i.e. $\tilde{a}$ = S | 0 … 0 | M:
    1. $M = 0 \Rightarrow \tilde{a} = (-1)^S 0$
    2. $M \neq 0 \Rightarrow \tilde{a} = (-1)^S \times 2^{-1022} \times 0.M$ (**subnormal range**)
2. $1 \leq E \leq 2046 \Rightarrow \tilde{a} = (-1)^S \times 2^{E-1023} \times 1.M$ (**normal range**)
3. $\texttt{E} = 2047$, i.e. $\tilde{a}$ = S | 1 … 1 | M:
    1. $M = 0 \Rightarrow \tilde{a} = (-1)^S \texttt{inf}$
    2. $M \neq 0 \Rightarrow \tilde{a} = \texttt{NaN}$(Not a Number) (**exceptions**)

See Figure 1.1 for a visual summary.

Examples:

- `realmin` is the smallest normalized positive machine number in FP64:

$$[\![\texttt{0|0 ... 01|0 ... 0}]\!]_{FP64} = 2^{1-1023} \times 1.0 = 2^{-1022}$$

`FP64` stands for IEEE **F**loating **P**oint **64** bit number representation. Whereas $[\cdot]_{\texttt{FP64}}$ is the `FP64` evaluation/interpration of the machine number

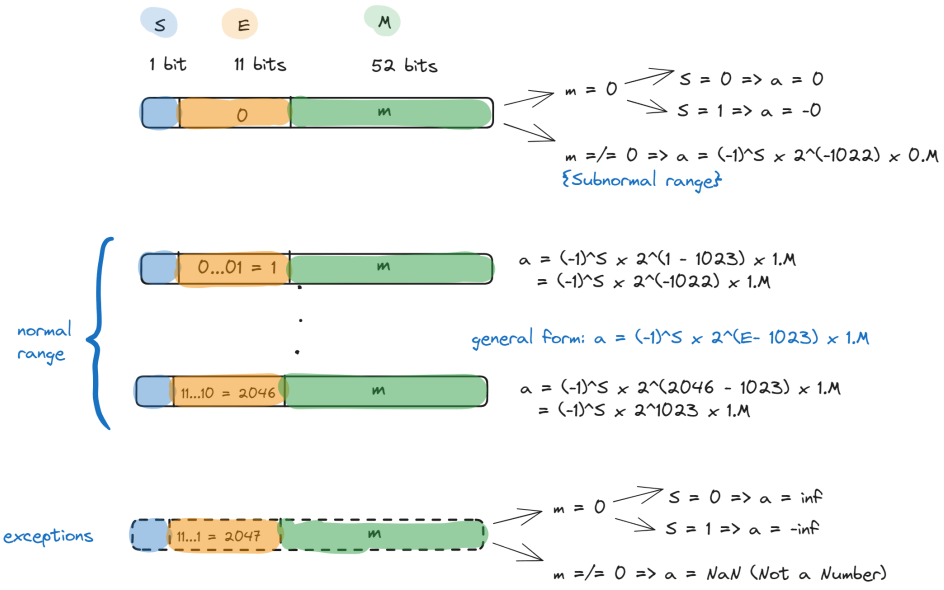

Figure 1.1: Evaluation of the IEEE 64 bit floating point numbers

- `realmax` is the greatest normalized machine number in FP64:

$$[0|1\ldots10|1\ldots1]_{\texttt{FP64}} \approx 1.7977\text{E}308$$

- $1 = 2^0 \times 1.0 = 2^{1023-1023} \times 1.0 = [0|01\ldots1|0\ldots0]_{\texttt{FP64}}$
- `eps` is defined as the spacing in the interval $(1, 2)$. Note that the spacing is constant for each such interval, but grows as we go further down the number line. That is, the spacing in $(1000, 1001)$ is also constant, but larger.
- number right after 1 is $[0|01\ldots1|0\ldots1]_{\texttt{FP64}}$. Then the spacing, i.e. `eps` in the above definition is 2^{-52}